

2025-DSE MATHEMATICS Compulsory Part Paper 2 Suggested Solution

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|-----|---|-----|---|
| 1. | C | 26. | B |
| 2. | B | 27. | A |
| 3. | D | 28. | C |
| 4. | A | 29. | A |
| 5. | A | 30. | B |
| 6. | D | 31. | D |
| 7. | D | 32. | C |
| 8. | C | 33. | C |
| 9. | D | 34. | D |
| 10. | C | 35. | A |
| 11. | A | 36. | B |
| 12. | C | 37. | A |
| 13. | A | 38. | A |
| 14. | A | 39. | B |
| 15. | D | 40. | B |
| 16. | B | 41. | C |
| 17. | B | 42. | B |
| 18. | C | 43. | C |
| 19. | D | 44. | A |
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| 21. | B | | |
| 22. | D | | |
| 23. | C | | |
| 24. | C | | |
| 25. | B | | |

SECTION A

1. C

$$\begin{aligned} & \frac{(27x)^5}{(3x^{-2})^4} \\ &= \frac{3^{15}x^5}{3^4x^{-8}} \\ &= 3^{11}x^{13} \end{aligned}$$

2. B

$$\begin{aligned} & 36 - (3m + 4n)^2 \\ &= 6^2 - (3m + 4n)^2 \\ &= (6 + 3m + 4n)(6 - 3m - 4n) \end{aligned}$$

3. D

$$\begin{aligned} & (x + 8)(x + a) + b \equiv x^2 + 5a(x + 3) \\ & x^2 + 8x + ax + 8a + b \equiv x^2 + 5ax + 15a \\ & \text{By comparing the like terms on both sides,} \\ & 8 + a = 5a \\ & a = 2 \\ & 8a + b = 15a \\ & b = 7a \\ & = 7(2) \\ & = 14 \end{aligned}$$

4. A

$$\begin{aligned} & (3c + 1)(d - 4) = 2d(5c - 1) \\ & 3cd - 12c + d - 4 = 10cd - 2d \\ & -7cd - 12c = 4 - 3d \\ & -c(7d + 12) = 4 - 3d \\ & c = \frac{3d - 4}{7d + 12} \end{aligned}$$

5. A

$$x^2 + 4x = k^2 - 2k - 3$$

$$x(x+4) = (k-3)(k+1)$$

For $x = k - 3$, For $x = -k - 1$,

$\begin{aligned} & x(x+4) \\ &= (k-3)(k-3+4) \\ &= (k-3)(k+1) \end{aligned}$	$\begin{aligned} & x(x+4) \\ &= (-k-1)(-k-1+4) \\ &= (-k-1)(-k+3) \\ &= (k+1)(k-3) \end{aligned}$
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Alternative solution:

$\begin{aligned} & x^2 + 4x = k^2 - 2k - 3 \\ & x^2 + 4x = (k-3)(k+1) \\ & x^2 + 4x - (k-3)(k+1) = 0 \\ & (x+k+1)[x-(k-3)] = 0 \\ & x = -k-1 \text{ or } x = k-3 \end{aligned}$	$\begin{array}{r} x \quad (k+1) \\ x \quad -(k-3) \\ \hline (k+1)x \quad -(k-3)x = 4x \end{array}$
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6. D

Since x is corrected to 2 decimal places, the range of values of x has 3 decimal places.

$$5.665 = 5.67 \text{ (correct to 2 decimal places)}, \quad 5.675 = 5.68 \text{ (correct to 2 decimal places)}$$

$$\therefore 5.665 \leq x < 5.675$$

Alternative solution:

$$\begin{aligned} 5.67 - \frac{0.01}{2} &\leq x < 5.67 + \frac{0.01}{2} \\ 5.665 &\leq x < 5.675 \end{aligned}$$

7. D

$$4y+1 < 5y-3 \text{ and } 5y-3 \leq 8y-9$$

$$4 < y \quad 6 \leq 3y$$

$$y > 4 \quad y \geq 2$$

$$\therefore y > 4$$

8. C

$$f(4) + f(-4) = 38$$

$$4^2 + 7(4) + k + (-4)^2 + 7(-4) + k = 38$$

$$2k = 6$$

$$k = 3$$

9. D

$$\begin{aligned} p(-3) &= 0 \\ n(-3)^3 - 3n(-3) + 36 &= 0 \\ -27n + 9n + 36 &= 0 \\ n &= 2 \\ p(x) &= 2x^3 - 6x + 36 \\ p(3) &= 2(3)^3 - 6(3) + 36 \\ &= 72 \end{aligned}$$

10. C

$$\begin{aligned} &40\,000 \times \left(1 + \frac{3\%}{2}\right)^{5 \times 2} \\ &= 46\,421.633 \\ &= \$46\,422 \text{ (corr. to the nearest dollar)} \end{aligned}$$

11. A

$$\begin{aligned} \frac{\alpha + 2\beta}{\beta + 2\gamma} &= \frac{4}{9} & \frac{\alpha + 2\beta}{\gamma + 2\alpha} &= \frac{4}{5} \\ 9\alpha + 18\beta &= 4\beta + 8\gamma & 5\alpha + 10\beta &= 4\gamma + 8\alpha \\ 9\alpha + 14\beta &= 8\gamma & -3\alpha + 10\beta &= 4\gamma \\ \therefore 9\alpha + 14\beta &= -6\alpha + 20\beta \\ 15\alpha &= 6\beta \\ \frac{\alpha}{\beta} &= \frac{2}{5} \end{aligned}$$

12. C

$$\begin{aligned} z &= \frac{kx^3}{y^2} \\ 3 &= \frac{k(3)^3}{(6)^2} \\ k &= 4 \\ z &= \frac{4(5)^3}{(2)^2} \\ &= 125 \end{aligned}$$

13. A

$$a_5 = 2a_4 + a_3$$

$$41 = 2a_4 + a_3 \dots (1)$$

$$a_4 = 2a_3 + a_2$$

$$a_4 = 2a_3 + 3 \dots (2)$$

Sub. (2) into (1) ,

$$41 = 2(2a_3 + 3) + a_3$$

$$a_3 = 7$$

$$a_4 = 2(7) + 3 = 17$$

$$a_6 = 2a_5 + a_4$$

$$= 2(41) + 17$$

$$= 99$$

14. A

Consider the x -intercept,

$$px + q(0) = 7$$

$$x = \frac{7}{p}$$

$$\therefore \frac{7}{p} < 1$$

$$p > 7$$

$\therefore$ I is true.

Consider the y -intercept,

$$p(0) + qy = 7$$

$$y = \frac{7}{q}$$

$$\therefore \frac{7}{q} > 1$$

$$q < 7$$

$\therefore$ II is not true.

$$\therefore p > q$$

$\therefore$ III is not true.

15. D

Consider the perimeter of the sector OMN ,

$$2r + MN = 12\pi$$

$$2(3\pi) + MN = 12\pi$$

$$MN = 6\pi$$

$$2\pi r \times \frac{\theta}{360^\circ} = 6\pi$$

$$2\pi(3\pi) \times \frac{\theta}{360^\circ} = 6\pi$$

$$\theta = \frac{360^\circ}{\pi}$$

∴ III is true.

$$\begin{aligned} \text{Area of the sector} &= \pi r^2 \times \frac{\theta}{360^\circ} \\ &= \pi(3\pi)^2 \times \frac{360^\circ}{\pi} \\ &= 9\pi^2 \text{ cm}^2 \end{aligned}$$

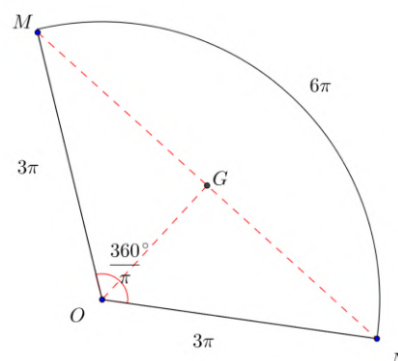
∴ I is true.

$$MN = 2NG$$

$$\begin{aligned} &= 2(3\pi) \sin\left(\frac{1}{2} \times \frac{360^\circ}{\pi}\right) \\ &= 15.86135438... \text{cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{The perimeter of } \triangle OMN &= 3\pi + 3\pi + MN \\ &= 34.71091031... \text{cm} \\ &< 35 \text{ cm} \end{aligned}$$

∴ II is true.



Alternative solution:

By cosine formula,

$$MN^2 = OM^2 + ON^2 - 2(OM)(ON)\cos\theta$$

$$MN^2 = (3\pi)^2 + (3\pi)^2 - 2(3\pi)(3\pi)\cos\frac{360^\circ}{\pi}$$

$$MN = 15.86135438... \text{cm}$$

16. B

Let the radius of the cylinder be r cm .

Consider the total surface area of the circular cylinder,

$$2\pi r^2 + 2\pi r(35) = 492\pi$$

$$2r^2 + 70r - 492 = 0$$

$$r = 6 \text{ or } r = -41 \text{ (rej.)}$$

The volume of the sphere

$$= \frac{4}{3}\pi(6)^3$$

$$= 288\pi \text{ cm}^3$$

17. B

$$DG : CG = 2 : 1$$

$$AE : BE = 1 : 3$$

Let $DG = 8h$ and $CG = 4h$.

Let $AE = 3h$ and $BE = 9h$.

Let I be a point on DC such that $EI \parallel AD \parallel BC$.

So, $DI = 3h$ and $IG = 5h$.

$$\frac{\text{The area of } \triangle EGI}{\text{The area of } \triangle HGC} = \left(\frac{IG}{CG}\right)^2$$

$$\frac{\text{The area of } \triangle EGI}{16} = \left(\frac{5h}{4h}\right)^2$$

$$\text{The area of } \triangle EGI = 25 \text{ cm}^2$$

$$\frac{\text{The area of } \triangle DEI}{\text{The area of } \triangle EGI} = \frac{DI}{GI}$$

$$\frac{\text{The area of } \triangle DEI}{25} = \frac{3h}{5h}$$

$$\text{The area of } \triangle DEI = 15 \text{ cm}^2$$

Note that the area of $\triangle AED = \text{the area of } \triangle DEI = 15 \text{ cm}^2$.

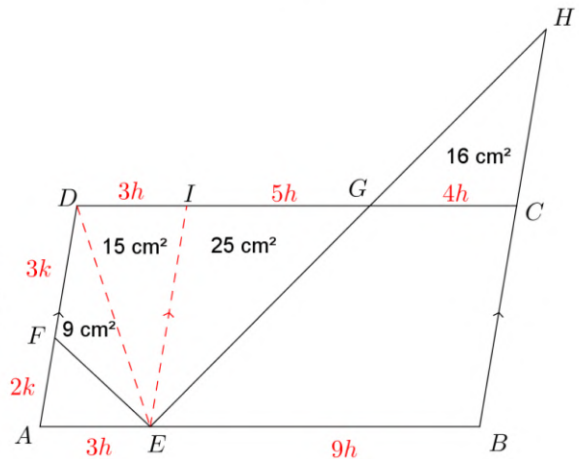
Let $AF = 2k$ and $DF = 3k$.

The area of $\triangle DFE$

$$= 15 \times \frac{3k}{2k + 3k}$$

$$= 9 \text{ cm}^2$$

$$\text{The area of quadrilateral } DFEG = 25 + 15 + 9 = 49 \text{ cm}^2$$



Alternative solution:

$$DG : CG = 2 : 1$$

$$AE : BE = 1 : 3$$

$$\text{Let } DG = 8h \text{ and } CG = 4h .$$

$$\text{Let } AE = 3h \text{ and } BE = 9h .$$

$$\therefore \triangle HGC \sim \triangle HEB \text{ (AAA)}$$

$$\begin{aligned} \text{Area of } GEBC &= 16 \times \left(\frac{9h}{4h} \right)^2 - 16 \\ &= 65 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \frac{\text{The area of } \triangle DAE}{\text{The area of } \triangle FAE} &= \frac{DA}{FA} \\ \frac{\text{The area of } \triangle DAE}{\text{The area of } \triangle FAE} &= \frac{3+2}{2} \\ &= \frac{5}{2} \end{aligned}$$

Since $\triangle DAE$ and $\triangle FAE$ have the same base AE ,

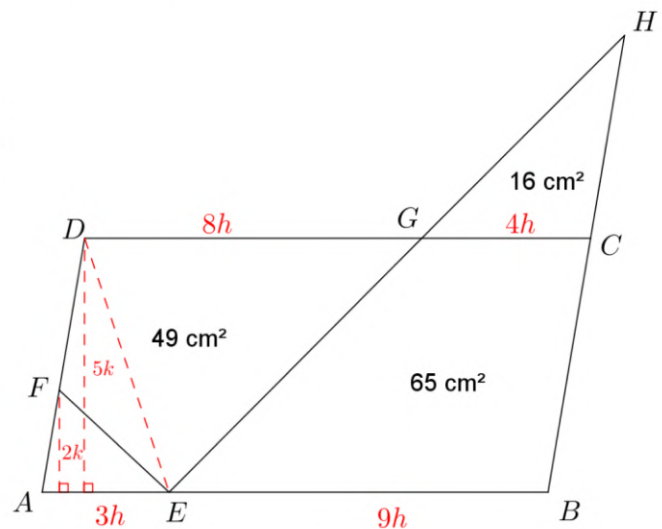
The altitude from D to AE : the altitude from F to $AE = 5 : 2$

Let the altitude from D to $AE = 5k$.

Let the altitude from F to $AE = 2k$.

$$\begin{aligned} \frac{(4h+9h)(5k)}{2} &= 65 \\ hk &= 2 \end{aligned}$$

$$\begin{aligned} \text{The area of } DFEG &= \frac{(3h+8h)(5k)}{2} - \frac{(3h)(2k)}{2} \\ &= \frac{49hk}{2} \\ &= 49 \text{ cm}^2 \end{aligned}$$



18. C

Note that $\angle WZX = 90^\circ$ (converse of Pyth. thm.)

Consider $\triangle XYZ$,

$$y^2 + 60^2 = x^2 \text{ (Pyth. thm.)}$$

Consider $WY : XY = XY : YZ$,

$$\frac{WY}{XY} = \frac{XY}{YZ}$$

$$\frac{25+y}{x} = \frac{x}{y}$$

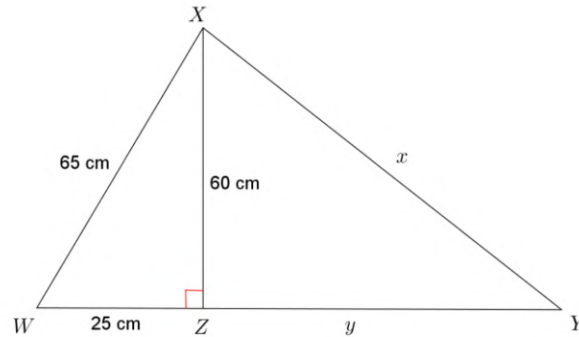
$$25y + y^2 = x^2$$

$$25y + y^2 = 60^2 + y^2$$

$$y = 144 \text{ cm}$$

$$x^2 = 60^2 + 144^2$$

$$x = 156 \text{ cm}$$



19. D

$\triangle DFC \cong \triangle BFC$ (SAS)

$DF = FB$ (corr. sides, $\cong \Delta$ s)

$FG = FB$ (sides opp. equal $\angle$ s)

$\therefore DF = FG$

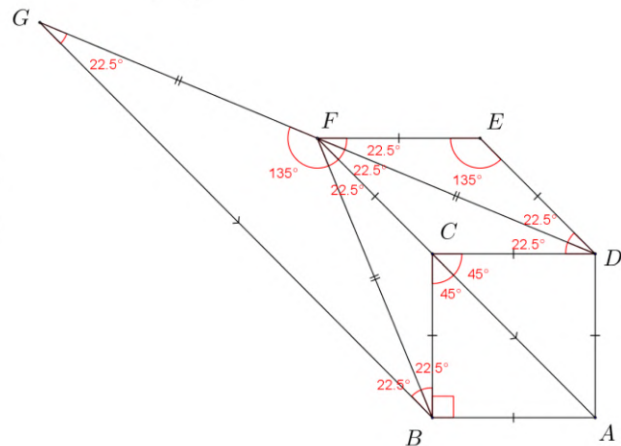
$\therefore$ I is true.

$\triangle BFG \sim \triangle DEF$ (AAA)

$\therefore$ II is true.

$$\begin{aligned} & \angle ABG + \angle BFD \\ &= (90^\circ + 22.5^\circ + 22.5^\circ) + (22.5^\circ + 22.5^\circ) \\ &= 180^\circ \end{aligned}$$

$\therefore$ III is true.



20. D

Let T be a point on PQ such that $PS \parallel TR$.

$$\angle TRS + \angle PSR = 180^\circ \text{ (int. } \angle\text{s, } PS \parallel TR)$$

$$\angle TRS + 120^\circ = 180^\circ$$

$$\angle TRS = 60^\circ$$

$$\angle TRQ = \angle QRS - \angle TRS$$

$$= 150^\circ - 60^\circ$$

$$= 90^\circ$$

In $\triangle TRQ$,

$$\frac{TR}{TQ} = \cos \angle QTR$$

$$\frac{41}{TQ} = \cos 60^\circ$$

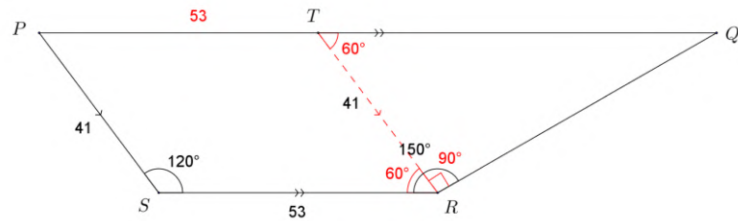
$$TQ = 82 \text{ cm}$$

PQ

$$= PT + TQ$$

$$= 53 + 82$$

$$= 135 \text{ cm}$$



21. B

Consider the area of $\triangle ADE$,

$$\frac{20 \times ED}{2} = 150$$

$$ED = 15 \text{ cm}$$

$$\tan \angle ADE = \frac{AE}{ED}$$

$$\tan \angle ADE = \frac{20}{15}$$

$$\angle ADE = 53.13010235...^\circ$$

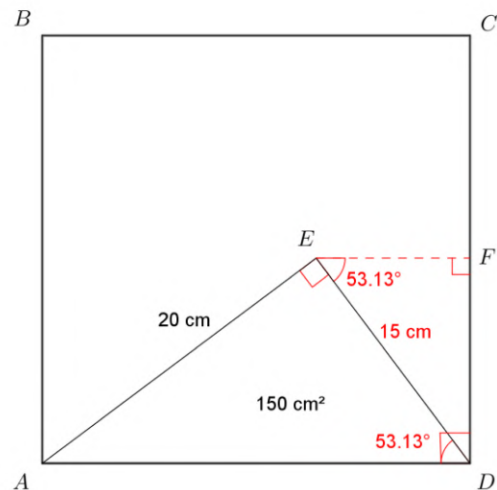
Let F be a point on CD such that $EF \perp CD$.

In $\triangle DEF$,

$$\frac{EF}{ED} = \cos \angle DEF$$

$$EF = 15 \cos 53.13010235...^\circ$$

$$= 9 \text{ cm}$$



22. D

Let $\angle SRT = x$.

Since RT is the angle bisector of $\angle SRU$, then

$$\angle URT = \angle SRT = x$$

$$\angle VUR = \angle URT = x \text{ (alt. } \angle\text{s, } RT \parallel VU\text{)}$$

$$\angle SUT = x \text{ (}\angle\text{s in the same segment)}$$

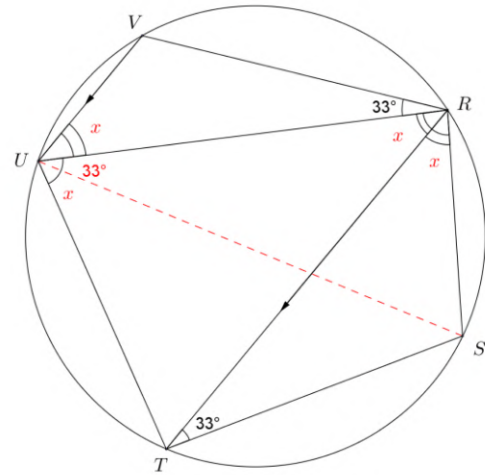
$$\angle SUR = 33^\circ \text{ (}\angle\text{s in the same segment)}$$

$$\angle VUT + \angle VRT = 180^\circ \text{ (opp. } \angle\text{s, cyclic quad.)}$$

$$x + x + 33^\circ + x + 33^\circ = 180^\circ$$

$$x = 38^\circ$$

$$\begin{aligned} \angle RUT &= 33^\circ + 38^\circ \\ &= 71^\circ \end{aligned}$$



23. C

Let $\angle ADC = x$, then $\angle ABC = 90^\circ - x$.

$$\angle ACB + \angle CAB + \angle ABC = 180^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}$$

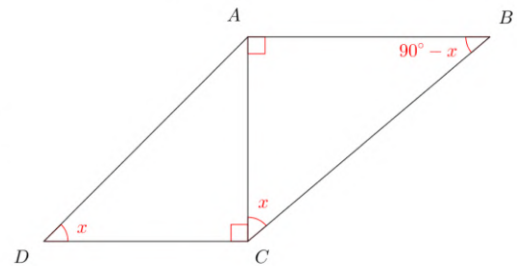
$$\angle ACB + 90^\circ + 90^\circ - x = 180^\circ$$

$$\angle ACB = x$$

$$\frac{AC}{BC} = \cos x \quad \frac{CD}{AD} = \cos x$$

$$AC = BC \cos x \quad CD = AD \cos x$$

$$\begin{aligned} \tan \angle ADC &= \frac{AC}{CD} \\ &= \frac{BC \cos x}{AD \cos x} \\ &= \frac{BC}{AD} \end{aligned}$$



24. C

Note that $\triangle OXY$ is a right-angled triangle.

$$OY^2 = OX^2 + XY^2 \quad (\text{Pyth. thm.})$$

$$2^2 = 1^2 + XY^2$$

$$XY = \sqrt{3}$$

$$YZ = XY = \sqrt{3}$$

$$\angle OYX$$

$$= 180^\circ - 90^\circ - 60^\circ$$

$$= 30^\circ$$

$$\angle OYZ$$

$$= 30^\circ + 60^\circ$$

$$= 90^\circ$$

$$OZ^2 = OY^2 + YZ^2 \quad (\text{Pyth. thm.})$$

$$OZ^2 = 2^2 + (\sqrt{3})^2$$

$$OZ = \sqrt{7}$$

$$r = \sqrt{7}$$

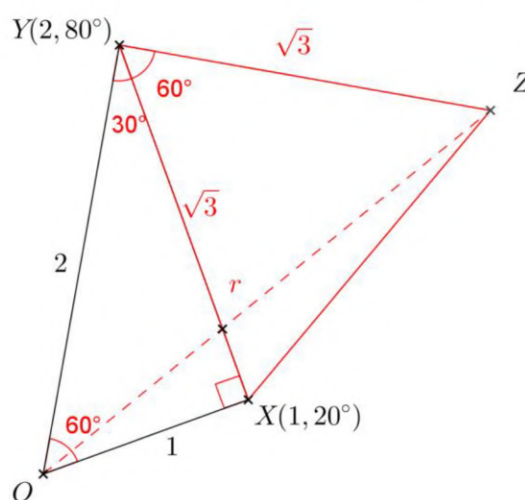
Alternative solution:

Consider $\triangle OXY$, by cosine formula,

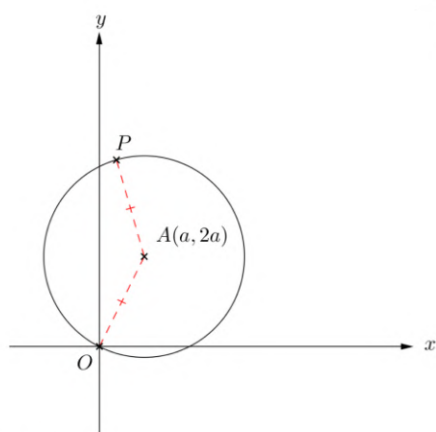
$$XY^2 = OX^2 + OY^2 - 2(OX)(OY)\cos \angle XOY$$

$$XY^2 = 1^2 + 2^2 - 2(1)(2)\cos 60^\circ$$

$$XY = \sqrt{3}$$



25. B



26. B

Let L_1 intersects the y -axis at point D .

Note that $\triangle AOD \sim \triangle ACB$ (AAA).

$$\left(\frac{AD}{AB} \right)^2 = \frac{\text{The area of } \triangle AOD}{\text{The area of } \triangle ACB}$$

$$\left(\frac{\sqrt{\left(\frac{20}{3} - 0 \right)^2 + (0 - 5)^2}}{\frac{20}{3} - \frac{20}{m}} \right)^2 = \frac{\frac{1}{2} \times 5 \times \frac{20}{3}}{6}$$

$$\frac{\frac{25}{3}}{\frac{20}{3} - \frac{20}{m}} = \frac{5}{3}$$

$$\frac{25m}{20m - 60} = \frac{5}{3}$$

$$100m - 300 = 75m$$

$$m = 12$$

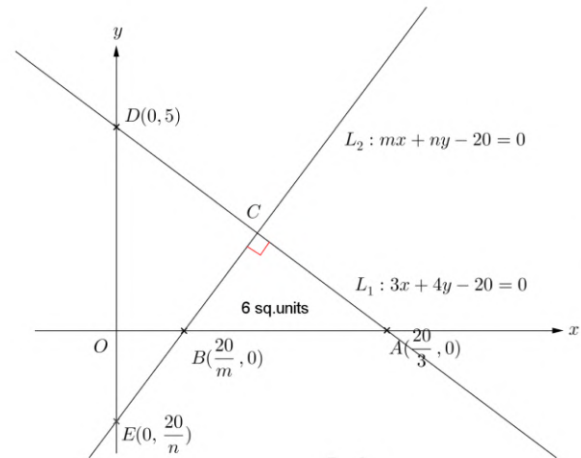
Slope of $L_2 \times$ slope of $L_1 = -1$

$$-\frac{m}{n} \times \left(-\frac{3}{4} \right) = -1$$

$$-\frac{m}{n} = \frac{4}{3}$$

$$-\frac{12}{n} = \frac{4}{3}$$

$$n = -9$$



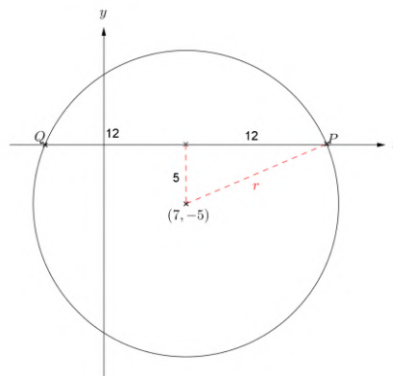
27. A

$$r^2 = 5^2 + \left(\frac{24}{2} \right)^2 \text{ (Pyth. thm.)}$$

$$r = 13$$

$$(x - 7)^2 + (y + 5)^2 = 13^2$$

$$x^2 + y^2 - 14x + 10y - 95 = 0$$



28. C

The expected number of tokens got in the game

$$= \frac{3}{6} \times 10 + \frac{1}{6} \times 15 + \frac{1}{6} \times 25 + \frac{1}{6} \times 50$$

$$= 20$$

29. A

The total number of teachers = 22 .

The lower quartile and the upper quartile are 6th datum and the 17th datum respectively.

Thus, the inter-quartile range = 5 - 4 = 1 .

30. B

Consider the mean of the above data,

$$\frac{\alpha + \beta + (-4) + (-3) + 1 + 1 + 1 + 4}{8} = 0$$

$$\alpha + \beta = 0$$

$\therefore$ III is true.

Since the range of the above data is 10 ,

$\therefore \alpha$ and β must be either 5 or -5.

Therefore, the above data become

-5 -4 -3 1 1 1 4 5

The mode is 1 . ($s = 1$)

$\therefore$ I is true.

The median is 1 . ($t = 1$)

$\therefore$ II is not true.

SECTION B

31. D

$$\begin{aligned}
 & 3E0000000000000_{16} \\
 &= 3 \times 16^{13} + 14 \times 16^{12} \\
 &= (2+1) \times (2^4)^{13} + (8+4+2) \times (2^4)^{12} \\
 &= 2 \times 2^{52} + 1 \times 2^{52} + 2^3 \times 2^{48} + 2^2 \times 2^{48} + 2 \times 2^{48} \\
 &= 2^{53} + 2^{52} + 2^{51} + 2^{50} + 2^{49}
 \end{aligned}$$

32. C

$$\begin{aligned}
 p^2 - 4q^2 &= (p-2q)(p+2q) & p^3 - 8q^3 &= (p-2q)(p^2 + 2pq + 4q^2) & (p+2q)(p^2 - 4q^2) &= (p+2q)^2(p-2q)
 \end{aligned}$$

$$\begin{aligned}
 & \text{L.C.M.} \\
 &= (p+2q)^2(p-2q)(p^2 + 2pq + 4q^2) \\
 &= (p+2q)^2(p^3 - 8q^3)
 \end{aligned}$$

33. C

The slope of the linear function

$$\begin{aligned}
 &= \frac{0-12}{2-0} \\
 &= -6
 \end{aligned}$$

The equation of the linear function is

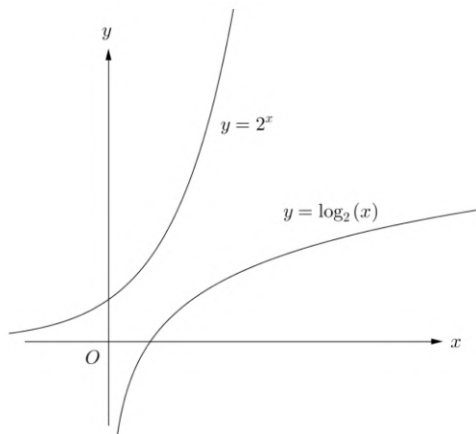
$$\begin{aligned}
 \log_5 y &= -6 \log_{25} x + 12 \\
 \log_5 y + 6 \log_{25} x &= 12 \\
 \log_5 y + \frac{\log_5 x^6}{\log_5 25} &= 12 \\
 \log_5 y + \frac{\log_5 x^6}{2} &= 12 \\
 2 \log_5 y + \log_5 x^6 &= 24 \\
 \log_5 y^2 x^6 &= 24 \\
 y^2 x^6 &= 5^{24} \\
 y &= 5^{12} x^{-3}
 \end{aligned}$$

$$\therefore n = -3$$

34. D

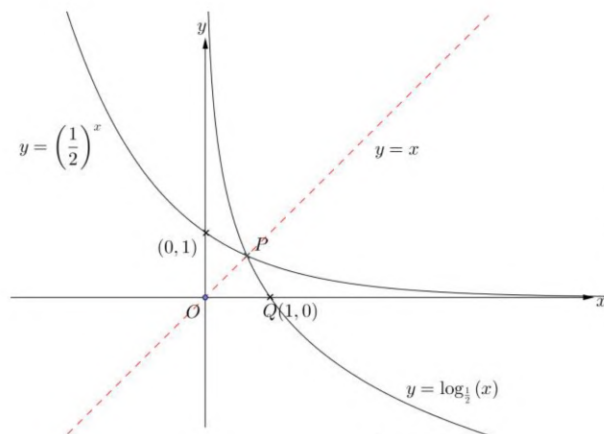
For $a > 1$,

for example $a = 2$,



For $0 < a < 1$,

for example $a = \frac{1}{2}$,



$\therefore$ I is true.

Q is the point of intersection of $y = \log_a x$ and the x -axis.

When $y = 0$, $x = 1$.

$\therefore OQ = 1$

$> a$

$\therefore$ II is true.

Since $y = a^x$ and $y = \log_a x$ are symmetric to each other about the straight line $y = x$,

P is a point lying on $y = x$.

$\tan \angle POQ = \text{slope of the straight line } y = x$

$\tan \angle POQ = 1$

$\angle POQ = 45^\circ$

$\therefore$ III is true.

(This question is controversial and may be cancelled.)

35. A

$$\begin{aligned} & i^9 + i^{10} + i^{11} + \cdots + i^{999} \\ &= (i^9 + i^{10} + i^{11} + i^{12}) + (i^{13} + i^{14} + i^{15} + i^{16}) + \cdots + (i^{993} + i^{994} + i^{995} + i^{996}) + i^{997} + i^{998} + i^{999} \\ &= (i - 1 - i + 1) + (i - 1 - i + 1) + \cdots + (i - 1 - i + 1) + i - 1 - i \\ &= -1 \end{aligned}$$

36. B

The points of intersections of the lines are $(1, 3)$, $(11, \frac{44}{3})$ and $(11, -5)$.

For $x=1$ and $y=3$,

$$\begin{aligned} & 8(1) - 6(3) + 11 \\ & = 1 \end{aligned}$$

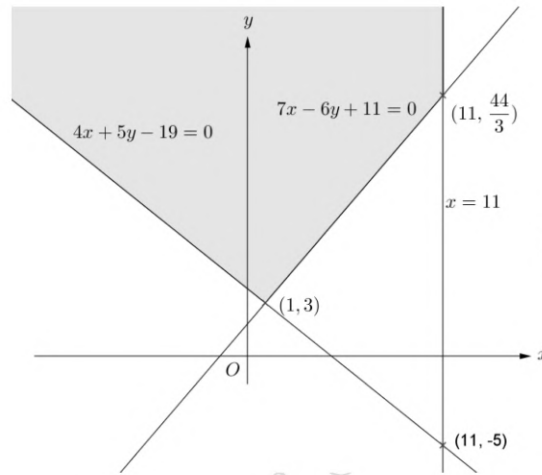
For $x=11$ and $y=\frac{44}{3}$,

$$\begin{aligned} & 8(11) - 6\left(\frac{44}{3}\right) + 11 \\ & = 11 \end{aligned}$$

For $x=11$ and $y=-5$,

$$\begin{aligned} & 8(11) - 6(-5) + 11 \\ & = 129 \end{aligned}$$

Since $(11, -5)$ does not lie in D , the greatest value of $8x - 6y + 11$ is 11.



37. A

$$\frac{3^r}{3^q} = 3^{r-q} \quad \frac{3^q}{3^p} = 3^{q-p}$$

$\therefore p, q, r$ is an arithmetic sequence.

$$\therefore r - q = q - p$$

$$\therefore 3^{r-q} = 3^{q-p}$$

$\therefore 3^p, 3^q, 3^r$ is a geometric sequence.

$\therefore$ I is true.

For $p=1, q=2$ and $r=3$,

$\frac{5}{1}, \frac{5}{2}, \frac{5}{3}$ is not a geometric sequence.

$\therefore$ II is not true.

For $p=1, q=2$ and $r=3$,

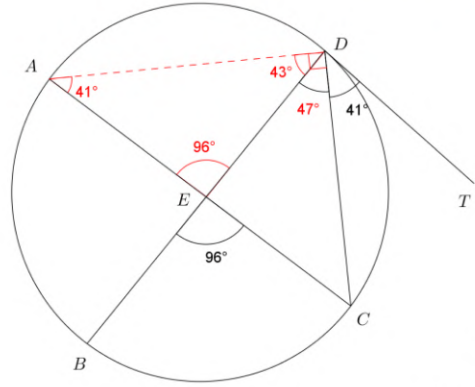
$$\begin{aligned} & \begin{array}{ccc} p-q & q-r & r-p \\ =1-2 & =2-3 & =3-1 \\ =-1 & =-1 & =2 \end{array} \end{aligned}$$

$-1, -1, 2$ is not an arithmetic sequence.

$\therefore$ III is not true.

38. A

$$\begin{aligned}\angle CAD &= 41^\circ \text{ (}\angle\text{s in alt. segment)} \\ \angle AED &= 96^\circ \text{ (vert. opp. } \angle\text{s)} \\ \angle ADE + 41^\circ + 96^\circ &= 180^\circ \text{ (}\angle \text{ sum of } \Delta) \\ \angle ADE &= 43^\circ \\ \angle ADC &= 90^\circ \text{ (}\angle \text{ in semi-circle)} \\ \angle CDE &= 90^\circ - 43^\circ \\ &= 47^\circ\end{aligned}$$



39. B

$$\begin{aligned}\tan^3 \theta &= 2 \tan \theta \\ \tan \theta (\tan^2 \theta - 2) &= 0 \\ \tan \theta &= 0 \quad \text{or} \quad \tan \theta = \pm \sqrt{2} \\ \theta &= 0^\circ \text{ (rej.) or } 180^\circ \text{ or } 360^\circ \text{ (rej.) or } 125^\circ \text{ or } 235^\circ\end{aligned}$$

40. B

Let T be a point on SR such that $QT \perp SR$.

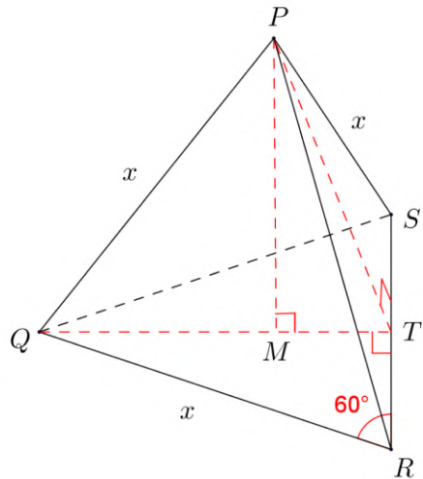
Let M be the projection of P on the plane QRS .

Since $PQRS$ is a regular tetrahedron, M lies on QT .

$$\begin{aligned}\frac{QT}{x} &= \sin 60^\circ \\ QT &= x \sin 60^\circ \\ PT &= QT \\ &= x \sin 60^\circ\end{aligned}$$

By cosine formula,

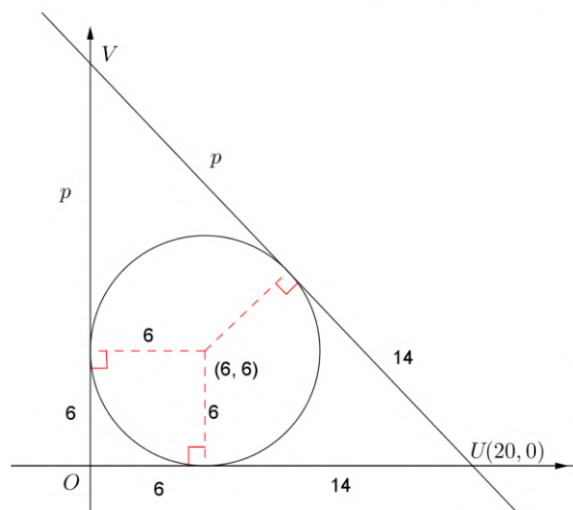
$$\begin{aligned}\cos \angle PQT &= \frac{PQ^2 + QT^2 - PT^2}{2(PQ)(QT)} \\ \cos \angle PQT &= \frac{x^2 + (x \sin 60^\circ)^2 - (x \sin 60^\circ)^2}{2(x)(x \sin 60^\circ)} \\ \cos \angle PQT &= \frac{x^2}{2x^2 \sin 60^\circ} \\ \angle PQT &= 54.73561032...^\circ\end{aligned}$$



41. C

$$\begin{aligned}(p+6)^2 + 20^2 &= (p+14)^2 \quad (\text{Pyth. thm.}) \\ p^2 + 12p + 36 + 400 &= p^2 + 28p + 196 \\ 240 &= 16p \\ p &= 15\end{aligned}$$

$$\begin{aligned}\text{The area of } \triangle OUV \\ &= \frac{20 \times (15 + 6)}{2} \\ &= 210\end{aligned}$$



42. B

$$\begin{aligned}\text{The required number} \\ &= C_7^{2+4+12} - C_7^{4+12} \\ &= 20\,384\end{aligned}$$

Alternative solution:

$$\begin{aligned}\text{The required number} \\ &= C_1^2 C_6^{4+12} + C_2^2 C_5^{4+12} \\ &= 20\,384\end{aligned}$$

43. C

$$\begin{aligned}\text{The required probability} \\ &= 1 - \frac{C_2^9 C_4^4}{C_6^{13}} \\ &= \frac{140}{143}\end{aligned}$$

44. A

$$\begin{aligned}\text{The difference of the standard score of the boy and girl} \\ &= \frac{6}{2} \\ &= 3 \\ \text{Thus,} \\ z &= -2 + 3 \quad \text{or} \quad z = -2 - 3 \\ &= 1 \quad \quad \quad = -5\end{aligned}$$

45. D

For $\{a, b, c, d\}$,

$$\text{mean} = m_1$$

$$\text{range} = r_1$$

$$\text{variance} = v_1$$

$$m_1 + m_3$$

$$= m_1 + m_1 + 3$$

$$= 2m_1 + 3$$

$$> m_2$$

$\therefore$ I is true.

For $\{2a, 2b, 2c, 2d\}$,

$$\text{mean} = m_2 = 2m_1$$

$$\text{range} = r_2 = 2r_1$$

$$\text{variance} = v_2 = 4v_1$$

$$r_1 + r_3$$

$$= r_1 + r_1$$

$$= 2r_1$$

$$= r_2$$

$\therefore$ II is true.

For $\{a+3, b+3, c+3, d+3\}$,

$$\text{mean} = m_3 = m_1 + 3$$

$$\text{range} = r_3 = r_1$$

$$\text{variance} = v_3 = v_1$$

$$v_1 + v_3$$

$$= v_1 + v_1$$

$$= 2v_1$$

$$< v_2$$

$\therefore$ III is true.