2025-DSE MATHEMATICS Compulsory Part Paper 1 Suggested Solution

SECTION A(1) (35 marks)

1. $$\frac{(x^4y^{-5})^3}{xy^2}$$
$$=\frac{x^{12}y^{-15}}{xy^2}$$
$$=\frac{x^{11}}{y^{17}}$$

2. $$\frac{1}{3d-4}-\frac{2}{6d+5}$$
$$=\frac{6d+5}{(3d-4)(6d+5)}-\frac{2(3d-4)}{(3d-4)(6d+5)}$$
$$=\frac{6d+5-6d+8}{(3d-4)(6d+5)}$$
$$=\frac{13}{(3d-4)(6d+5)}$$

3. We have $2m+3n=999$ and $\frac{m}{n}=\frac{8}{7}$.

$$\frac{m}{n}=\frac{8}{7}$$
$$m=\frac{8n}{7}$$

Put $m=\frac{8n}{7}$ into $2m+3n=999$.

$$2\left(\frac{8n}{7}\right)+3n=999$$
$$37n=6993$$
$$n=189$$

Alternative solution:

Let $m = 8k$ and $n = 7k$.

$$2(8k)+3(7k)=999$$
$$37k=999$$
$$k=27$$

Thus, $n = 189$.

4. (a) $(-2,-4)$

(b) The coordinates of point C are $(-2,-4+t)$.

Since A , O and C are collinear, slope of OC = slope of OA .

$$\frac{-4+t-0}{-2-0}=\frac{-2-0}{4-0}$$
$$-16+4t=4$$
$$t=5$$

5. (a) $10pr-6qr$
$=2r(5p-3q)$

(b) $25p^2-9q^2$
$=(5p-3q)(5p+3q)$

(c) $25p^2-9q^2-10pr+6qr$
$=(5p-3q)(5p+3q)-2r(5p-3q)$
$=(5p-3q)(5p+3q-2r)$

6. (a) $\frac{6x+1}{2}<x-8$ or $3x\le -21$

$6x+1<2x-16$ $\quad$ $x\le -7$

$4x<-17$

$x<-\frac{17}{4}$

Thus, the solution of (*) is $x<-\frac{17}{4}$.

(b) -5

7. (a) Let $\$m$ be the marked price of the souvenir.

$m(1-60\%)=378$
$m=945$

The marked price of the souvenir is \$945 .

(b) Let $\$C$ be the cost of the souvenir.

$C(1+75\%)=945$
$C=540$

The cost of the souvenir is \$540 .

(c) The selling price of the souvenir

$= \$378$

$< \$540$

The selling price of the souvenir is less than the cost of the souvenir.

Thus, there is a loss after selling the souvenir.

8. (a) $WS = WT$ (given)

$\angle UWS = \angle VWT$ (vert. opp. $\angle$s)

$\angle USW = \angle VTW$ (alt. $\angle$s, $SU // VT$)

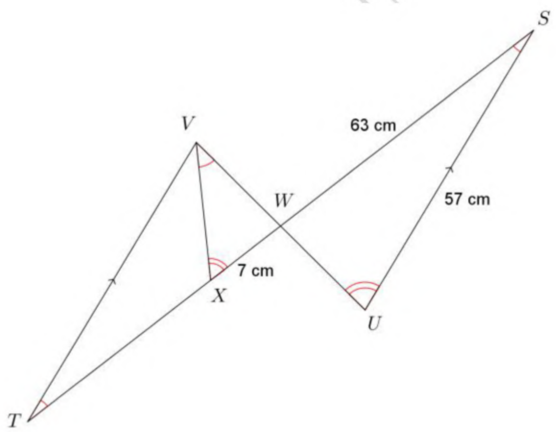
$\Delta SUW \cong \Delta TVW$ (ASA)

(b)(i) $\because \Delta SUW \cong \Delta TVW$

$WU = VW$ (corr. sides, $\cong \Delta$s)

$SU = TV$ (corr. sides, $\cong \Delta$s)

$\because \Delta SUW \sim \Delta VXW$



$\frac{VW}{SW} = \frac{WX}{WU}$ (corr. sides, $\sim\Delta$s)

$\frac{VW}{63} = \frac{7}{VW}$

$VW^2 = 441$

$VW = 21$ cm

$\frac{VX}{SU} = \frac{VW}{WS}$ (corr. sides, $\sim\Delta$s)

$\frac{VX}{57} = \frac{21}{63}$

$VX = 19$ cm

The perimeter of ΔTVX

$= TV + TX + VX$

$= 57 + (63 - 7) + 19$

$= 132$ cm

9. (a) For the least value of s,

$9 + 13 = 1 + s + 11$

$s = 10$

$\therefore$ The least possible value of s is 10 .

Alternative solution:

Since the median of the distribution is 7, we have

$9+13<2+s+11$ and $s+11<9+13+2$

$9<s$ $\qquad$ $s<13$

$\therefore 9<s<13$

Thus, the least possible value of s is 10 .

(b) 12

(c) For $s = 10$, the standard deviation of the distribution = 1.51 .

For $s = 11$, the standard deviation of the distribution = 1.50 .

For $s = 12$, the standard deviation of the distribution = 1.49 .

Thus, the greatest possible standard deviation of the distribution is 1.51 .

SECTION A(2) (35 marks)

10. (a)

$$\mathrm{f}(-2)=-45$$

$$2(-2)^3+h(-2)^2+k(-2)+15=-45$$

$$2h-k=-22$$

$$\mathrm{f}\left(\frac{5}{2}\right)=0$$

$$2\left(\frac{5}{2}\right)^3+h\left(\frac{5}{2}\right)^2+k\left(\frac{5}{2}\right)+15=0$$

$$5h+2k=-37$$

Solving, we have $h=-9$ and $k=4$.

(b)

$$\mathrm{f}(x)=0$$

$$2x^3-9x^2+4x+15=0$$

$$(2x-5)(x^2-2x-3)=0$$

$$(2x-5)(x-3)(x+1)=0$$

$$x=\frac{5}{2},\ x=3 \text{ or } x=-1$$

$\frac{5}{2}$, 3 and -1 are rational numbers.

Thus, the claim is agreed.

11. (a) Let $p(x)=ax+bx^2$, where a and b are non-zero constants.

$$p(7)=56$$
$$a(7)+b(7)^2=56$$
$$a+7b=8$$
$$p(9)=216$$
$$a(9)+b(9)^2=216$$
$$a+9b=24$$

Solving, we have $a=-48$ and $b=8$.

Thus, $p(x)=-48x+8x^2$.

(b) $$p(x)=c$$
$$8x^2-48x-c=0$$

Since $p(x)=c$ has two distinct real roots, $\Delta>0$.

$$(-48)^2-4(8)(-c)>0$$
$$32c>-2304$$
$$c>-72$$

12. (a) The range of the distribution = 42 kg

The inter-quartile range of the distribution
$=42-25$
$=17$ kg

Therefore, we have

$$69-(50+w)=17$$
$$w=2$$

(b)(i) $64-69=-5$ kg

The upper quartile of the distribution due to training decreases 5 kg .

(ii) The range of the distribution after the training
$=67-46$
$=21$ kg
<42 kg

The inter-quartile range of the distribution after the training
$=64-54$
$=10$ kg
<17 kg

Both the range and the inter-quartile range of the distribution decrease after the training. Thus, the distribution of weights of the athletes after the training is less dispersed than that before the training.

13. (a) Γ is the perpendicular bisector of MN.

(b) Let the coordinates of M and N be $(m, 0)$ and $(0, n)$ respectively.

The coordinates of the mid-point of MN are $\left(\dfrac{m}{2}, \dfrac{n}{2}\right)$.

Since Γ is the perpendicular bisector of MN, the mid-point of M and N lies on Γ.

$$3\left(\frac{m}{2}\right)-2\left(\frac{n}{2}\right)-30=0$$

$$3m-2n-60=0$$

slope of Γ $\times$ slope of $L=-1$

$$\frac{3}{2}\times\frac{n-0}{0-m}=-1$$

$$3n=2m$$

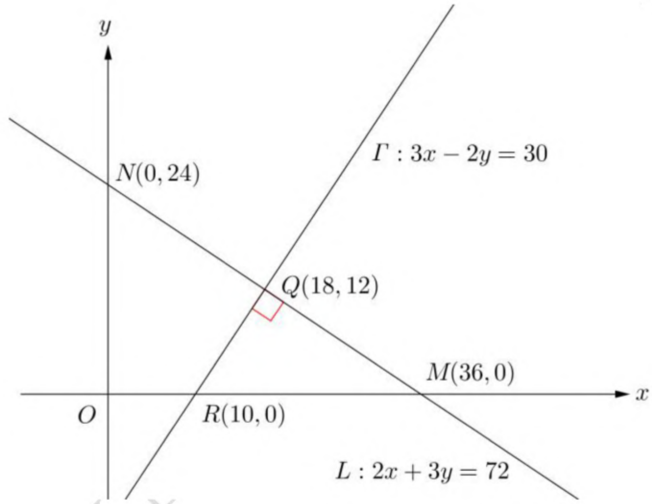
Solving, we have $m=36$ and $n=24$.

The equation of L is

$$y=\frac{24-0}{0-36}x+24$$

$$-3y=2x-72$$

$$2x+3y-72=0$$



(c) Note that the coordinates of Q are $(18, 12)$.

Note that the coordinates of R are $(10, 0)$.

The area of the quadrilateral $ONQR$

$=$ area of ΔOMN $-$ area of ΔMQR

$$=\frac{(36-0)\times(24-0)}{2}-\frac{(36-10)\times(12-0)}{2}$$

$=276$

14. (a) Let r cm be the radius of the upper base of the frustum.

$$\frac{r}{24}=\frac{45-30}{45}$$

$$r=8$$

The volume of X

$$=\frac{1}{3}\pi(24)^2(45)-\frac{1}{3}\pi(8)^2(45-30)$$

$=8\ 320\pi$ cm^3

Alternative solution:

The volume of X

$$=\frac{1}{3}\pi(24)^2(45)\left[1-\left(\frac{45-30}{45}\right)^3\right]$$

$=8\ 320\pi$ cm^3

(b) The total surface area of X

$= \pi(8)^2 + \pi(24)^2 + \pi(24)(\sqrt{24^2+45^2}) - \pi(8)\left[\sqrt{8^2+(45-30)^2}\right]$

$= 1\,728\pi \text{ cm}^2$

(c) Let x cm be the length of side of the cube.

$x^3 = 8\,320\pi$

$x = 29.67730087...$

The total surface area of the cube

$= 6x^2$

$= 6(29.67730087...)^2$

$= 5\,284.453123...$

$< 1\,728\pi$

Thus, the total surface area of the cube does not exceed the total surface area of X .

SECTION B (35 marks)

15. (a) The required probability

$= \dfrac{C_3^8 + C_3^4}{C_3^{13}}$

$= \dfrac{30}{143}$

(b) The required probability

$= \dfrac{C_1^8 \times C_1^4 \times C_1^1}{C_3^{13}}$

$= \dfrac{16}{143}$

16. (a) $\log_3 x + \log_3 y = 9$

$\log_3 x = 9 - u$

$\log_x 81 - \log_y 9 = 1$

$\dfrac{\log_3 81}{\log_3 x} - \dfrac{\log_3 9}{\log_3 y} = 1$

$\dfrac{4}{9-u} - \dfrac{2}{u} = 1$

$4u - 2(9-u) = u(9-u)$

$4u - 18 + 2u = 9u - u^2$

$u^2 - 3u - 18 = 0$

(b) $u^2-3u-18=0$

$(u-6)(u+3)=0$

$u=6$ or $u=-3$

$\log_3 y=6$ or $\log_3 y=-3$

Sub. $\log_3 y=6$ into $\log_3 x+\log_3 y=9$,

$\log_3 x+6=9$

$\log_3 x=3$

$x=27$

Sub. $\log_3 y=-3$ into $\log_3 x+\log_3 y=9$,

$\log_3 x+(-3)=9$

$\log_3 x=12$

$x=3^{12}$ (rejected, since $y=3^{-3}$ and x should be smaller than y)

Thus, $x=27$.

17. (a) Let d be the common difference of the arithmetic sequence.

$\mathrm{T}(47)=456$

$\mathrm{T}(1)+46d=456$

$\mathrm{T}(1)=456-46d$

Consider $\mathrm{T}(9),\mathrm{T}(47),\mathrm{T}(199)$ is a geometric sequence.

$$\frac{\mathrm{T}(47)}{\mathrm{T}(9)}=\frac{\mathrm{T}(199)}{\mathrm{T}(47)}$$

$$\frac{\mathrm{T}(1)+46d}{\mathrm{T}(1)+8d}=\frac{\mathrm{T}(1)+198d}{\mathrm{T}(1)+46d}$$

$$\left[\mathrm{T}(1)+46d\right]^2=\left[\mathrm{T}(1)+8d\right]\left[\mathrm{T}(1)+198d\right]$$

$$\left[\mathrm{T}(1)\right]^2+92d\,\mathrm{T}(1)+2\ 116d^2=\left[\mathrm{T}(1)\right]^2+206d\,\mathrm{T}(1)+1\ 584d^2$$

$$532d^2-114d\,\mathrm{T}(1)=0$$

$$d\left[14d-3(456-46d)\right]=0$$

$$d(152d-1\ 368)=0$$

$d=9$ or $d=0$ (rejected, since $\mathrm{T}(1)\neq\mathrm{T}(2)$)

$\mathrm{T}(1)=456-46(9)=42$

(b) $\dfrac{n[2a+(n-1)d]}{2} > 10^6$

$\dfrac{n[2(42)+(n-1)(9)]}{2} > 10^6$

$9n^2 + 75n - 2\times10^6 > 0$

$n < \dfrac{-75-\sqrt{75^2-4(9)(-2\times10^6)}}{2\times9}$ or $n > \dfrac{-75+\sqrt{75^2-4(9)(-2\times10^6)}}{2\times9}$

$n < -475.5896013...$ or $n > 467.256268...$

Thus, the least value of n is 468 .

18. (a) $g(x)$

$= 3x^2 - 6kx + 24x + 3k^2 - 24k + 55$

$= 3(x^2 - 2kx + 8x) + 3k^2 - 24k + 55$

$= 3\left[x^2 - 2(k-4)x + (k-4)^2 - (k-4)^2\right] + 3k^2 - 24k + 55$

$= 3[x-(k-4)]^2 - 3(k-4)^2 + 3k^2 - 24k + 55$

$= 3[x-(k-4)]^2 + 7$

Thus, the coordinates of R are $(k-4, 7)$.

(b)(i) Note that the coordinates of S are $(k+2, -3)$.

Let the coordinates of T be (x, y) .

Since O is the orthocentre of ΔRST ,

$OT \perp RS$ and $OR \perp TS$

$\dfrac{y-0}{x-0} \times \dfrac{7-(-3)}{k-4-(k+2)} = -1$

$\dfrac{y}{x} = \dfrac{6}{10}$

$y = \dfrac{3}{5}x$

$\dfrac{7-0}{k-4-0} \times \dfrac{y-(-3)}{x-(k+2)} = -1$

$7(y+3) = -[x-(k+2)](k-4)$

$7\left(\dfrac{3x}{5}+3\right) = -[(k-4)x-(k-4)(k+2)]$

$21x + 105 = -5(k-4)x + 5(k-4)(k+2)$

$[5(k-4)+21]x = 5(k-4)(k+2) - 105$

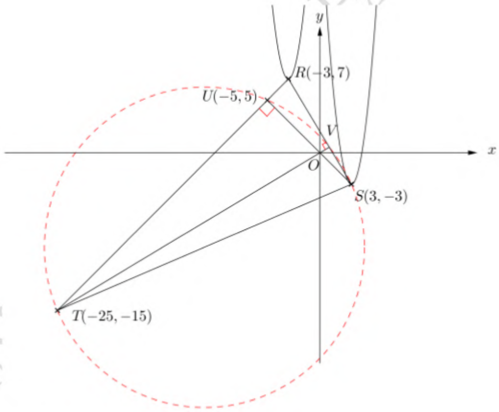
$x = \dfrac{5k^2 - 10k - 145}{5k+1}$

$$y = \frac{3}{5}\left(\frac{5k^2 - 10k - 145}{5k+1}\right)$$
$$= \frac{3k^2 - 6k - 87}{5k+1}$$

The coordinates of T are $\left(\frac{5k^2 - 10k - 145}{5k+1}, \frac{3k^2 - 6k - 87}{5k+1}\right)$.

(b)(ii) Since S, T, U and V are concyclic,

$\angle TUS = \angle TVS = 90°$ ($\angle$s in the same segment)



$\therefore$ The slope of TU $\times$ the slope of $US = -1$

$$\frac{\frac{3k^2 - 6k - 87}{5k+1} - 5}{\frac{5k^2 - 10k - 145}{5k+1} - (-5)} \times \frac{-3-5}{k+2-(-5)} = -1$$
$$\frac{3k^2 - 6k - 87 - 5(5k+1)}{5k^2 - 10k - 145 + 5(5k+1)} = \frac{k+7}{8}$$
$$\frac{3k^2 - 31k - 92}{5k^2 + 15k - 140} = \frac{k+7}{8}$$
$$5k^3 + 35k^2 + 15k^2 + 105k - 140k - 980 = 24k^2 - 248k - 736$$
$$5k^3 + 26k^2 + 213k - 244 = 0$$
$$(k-1)(5k^2 + 31k + 244) = 0$$

$\therefore k - 1 = 0$ or $5k^2 + 31k + 244 = 0$

$k = 1$ no real solutions

$\therefore k = 1$

19. (a) Since C passes through the point $(-10, 9)$, we have

$$(-10)^2 + (9)^2 - 10a - 2(9) + b = 0$$
$$b = 10a - 163$$

$$4x - 3y + 83 = 0$$
$$y = \frac{4x+83}{3}$$

Sub. $y = \frac{4x+83}{3}$ and $b = 10a - 163$ into $x^2 + y^2 + ax - 2y + b = 0$, we have

$$x^2+\left(\frac{4x+83}{3}\right)^2+ax-2\left(\frac{4x+83}{3}\right)+10a-163=0$$

$$9x^2+(4x+83)^2+9ax-6(4x+83)+90a-1467=0$$

$$9x^2+16x^2+664x+6889+9ax-24x-498+90a-1467=0$$

$$25x^2+(9a+640)x+90a+4924=0$$

Since L is a tangent to C, L and C have only one point of intersection.

Thus, the equation $25x^2+(9a+640)x+90a+4924=0$ has equal roots.

$$\Delta=0$$

$$(9a+640)^2-4(25)(90a+4924)=0$$

$$81a^2+2520a-82800=0$$

$$a=20 \text{ or } a=-\frac{460}{9} \text{ (rejected)}$$

$$b=37$$

(b)(i) Note that the equation of C is $x^2+y^2+20x-2y+37=0$.

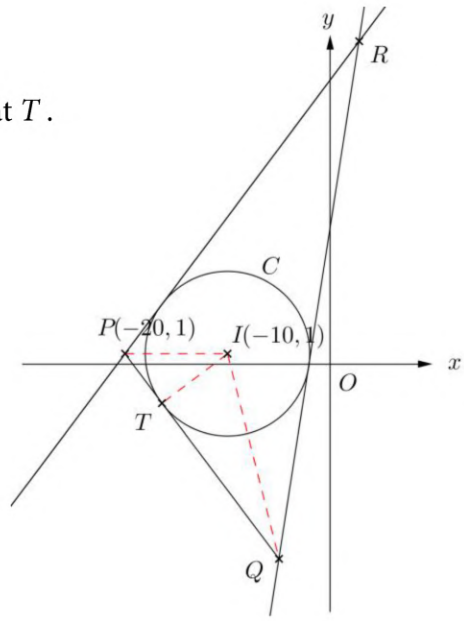
Therefore, the coordinates of I are $(-10, 1)$.

The radius of C $=\sqrt{(-10)^2+1^2-37}=8$

$$PI=-10-(-20)$$
$$=10$$

Let T be a point on PQ such that PQ is tangent to C at T .

Consider ΔIPT ,



$$\cos\angle IPT=\frac{PT}{IP}$$
$$=\frac{\sqrt{10^2-8^2}}{10}$$
$$=\frac{3}{5}$$

$$\therefore \cos\angle IPQ=\frac{3}{5}$$

(ii) Consider ΔIPQ , by cosine formula,

$$IQ^2=IP^2+PQ^2-2(IP)(PQ)\cos\angle IPQ$$
$$IQ^2=(10)^2+(25)^2-2(10)(25)\left(\frac{3}{5}\right)$$
$$IQ=\sqrt{425}$$
$$=5\sqrt{17}$$

Alternative solution:

$$TQ=25-\sqrt{10^2-8^2}=19$$
$$IQ=\sqrt{8^2+19^2} \text{ (Pyth. thm.)}$$
$$=\sqrt{425}$$
$$=5\sqrt{17}$$

(iii) $\sin\angle IQT = \dfrac{IT}{IQ}$

$$\sin\angle IQT = \frac{8}{5\sqrt{17}}$$

$$\angle IQT = 22.83365418...^\circ$$

$$\cos\angle IPT = \frac{3}{5}$$

$$\angle IPT = 53.13010235...^\circ$$

Since I is the in-centre of ΔPQR ,

$\angle IPR = \angle IPT = 53.13010235...^\circ$

$\angle IQR = \angle IQT = 22.83365418...^\circ$

In ΔPQR ,

$\angle PRQ = 180^\circ - 2\times(53.13010235...^\circ) - 2\times(22.83365418...^\circ)$ ($\angle$ sum of Δ)

$\angle PRQ = 28.07248694...^\circ$

By sine formula,

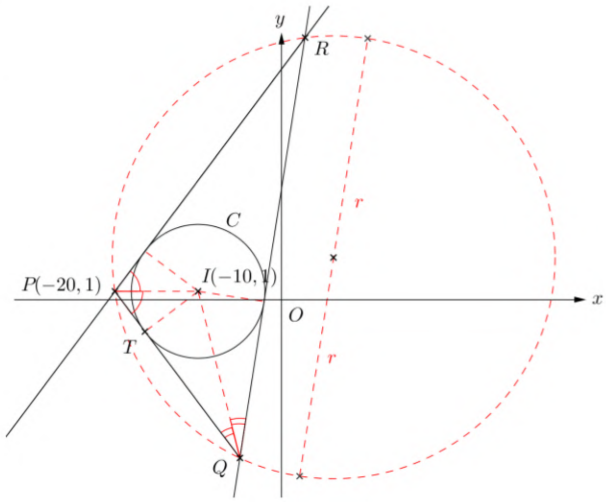
$$\frac{RQ}{\sin\angle QPR} = \frac{PQ}{\sin\angle PRQ}$$

$$\frac{RQ}{\sin(2\times 53.13010235...^\circ)} = \frac{25}{\sin 28.07248694...^\circ}$$

$$RQ = 51$$

Note that RQ is the longest side of ΔPQR .

$$\begin{aligned}\text{The diameter of the circle } &\geq RQ\\ 2r &\geq RQ\\ 2r &\geq 51\\ r &\geq \frac{51}{2}\\ &> PQ\end{aligned}$$



Thus, the claim is agreed.

Alternative solution 1:

$$\sin\angle IQT = \frac{IT}{IQ}$$

$$\sin\angle IQT = \frac{8}{5\sqrt{17}}$$

$$\angle IQT = 22.83365418...^\circ$$

$$\cos\angle IPT = \frac{3}{5}$$

$$\angle IPT = 53.13010235...^\circ$$

Since I is the in-centre of ΔPQR,

$$\angle IPR = \angle IPT = 53.13010235...^\circ$$

$$\angle IQR = \angle IQT = 22.83365418...^\circ$$

In ΔPQR,

$$\angle PRQ = 180^\circ - 2\times(53.13010235...^\circ) - 2\times(22.83365418...^\circ) \quad (\angle \text{ sum of } \Delta)$$

$$\angle PRQ = 28.07248694...^\circ$$

Let G be the centre of the circumcircle of ΔPQR.

$$\begin{aligned}\angle PGQ &= 2\times\angle PRQ \quad (\angle \text{ at centre twice } \angle \text{ at } \odot^{\text{ce}})\\ &= 2\times 28.07248694...^\circ\\ &= 56.14497387...^\circ\end{aligned}$$

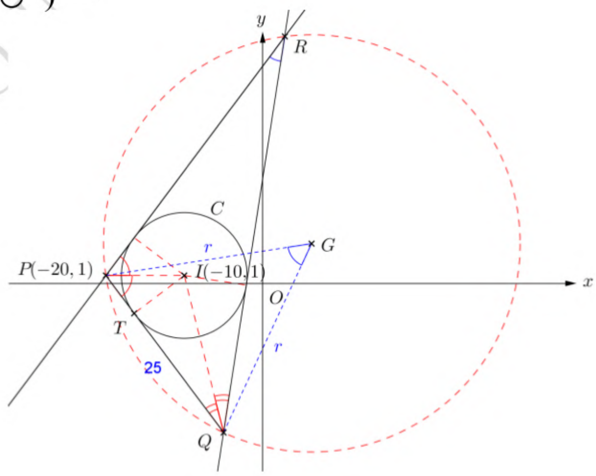
Consider ΔPQG, by cosine formula,

$$PQ^2 = PG^2 + GQ^2 - 2(PG)(QG)\cos\angle PGQ$$

$$25^2 = r^2 + r^2 - 2r^2\cos 56.14497387...^\circ$$

$$625 = r^2(2 - 2\cos 56.14497387...^\circ)$$

$$\begin{aligned} r &= 26.5625\\ &> PQ\end{aligned}$$



Thus, the claim is agreed.

Alternative solution 2:

$\sin\angle IQT = \dfrac{IT}{IQ}$

$\sin\angle IQT = \dfrac{8}{5\sqrt{17}}$

$\angle IQT = 22.83365418...^\circ$

$\cos\angle IPT = \dfrac{3}{5}$

$\angle IPT = 53.13010235...^\circ$

Since I is the in-centre of ΔPQR ,

$\angle IPR = \angle IPT = 53.13010235...^\circ$

$\angle IQR = \angle IQT = 22.83365418...^\circ$

In ΔPQR ,

$\angle PRQ = 180^\circ - 2\times(53.13010235...^\circ) - 2\times(22.83365418...^\circ)$ ($\angle$ sum of Δ)

$\angle PRQ = 28.07248694...^\circ$

Denote S be a point on the circumcircle of ΔPQR such that PS is a diameter.

$\angle PQS = 90^\circ$ ($\angle$ in semi-circle)

$\angle PSQ = \angle PRQ = 28.07248694...^\circ$ ($\angle$s in the same segment)

$\dfrac{PQ}{PS} = \sin\angle PSQ$

$\dfrac{25}{2r} = \sin 28.07248694...^\circ$

$r = 26.5625$

$> PQ$

Thus, the claim is agreed.